Watson · A Diagram of Navigation · 1822

DESCRIPTION AND USE

OF A

DIAGRAM OF NAVIGATION;

BY WHICH

ALL PROBLEMS IN PLANE, TRAVERSE, PARALLEL, MIDDLE LATITUDE AND MERCATOR'S

SAILING

MAY BE INSTANTLY AND ACCURATELY RESOLVED.

Adapted to the capacity of all who know the use of Figures.

Designed as an easy Introduction, by sensible Demonstration, to the Principles, and the Practice of Navigation.

WITH A DIAGRAM ENGRAVED.

BY GEORGE WATSON, ESQ.

BELFAST, MAINE:
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1822.

DISTRICT OF MAINE, ss.

BE IT REMEMBERED, That on this twenty-second day of June, in the year of our Lord one thousand eight hundred and twenty-two, and the forty-sixth year of the Independence of the United States of America, GEORGE WATSON, Esquire, of the District of Maine, has deposited in this Office, the title of a Book, the right whereof he claims as Author, in the words following, viz: "*Description and use of a Diagram of Navigation; by which all problems in Plane, Traverse, Parallel, Middle Latitude and Mercator's Sailing may be instantly and accurately resolved. Adapted to the capacity of all who know the use of figures. Designed as an easy introduction, by sensible demonstration, to the principles and the practice of Navigation. With a diagram engraved.*" In conformity to the Act of the Congress of the United States, entitled, "An Act for the encouragement of learning, by securing the copies of maps, charts and books, to the authors and proprietors of such copies, during the times therein mentioned;" and also to an act, entitled, "An Act supplementary to an act, entitled, an act for the encouragement of learning, by securing the copies of maps, charts, and books, to the authors and proprietors of such copies, during the times therein mentioned, and extending the benefits thereof to the arts of designing, engraving, and etching historical and other prints."

JOHN MUSSEY, Jun. *Clerk of the District Court of Maine.*

A true copy as of record—Attest,

JOHN MUSSEY, Jun. *Clerk D. C. M.*

PREFACE.

Since the discovery of the western continent by Columbus, in 1492, the Art of Navigation has excited the attention of all maritime nations. Among the able writers who have investigated this great and interesting subject, none are more eminent than our distinguished countryman Dr. Bowditch, author of the *New American Practical Navigator*. The Diagram of Navigation is not to be considered a substitute for that standard work; on the contrary its object is to render that work more extensively useful, by exciting the attention of all persons to the general subject of Navigation, by a system the most easy and comprehensive; and especially that every seaman who can read and write, and who may know the use of figures in the four simple rules of arithmetic, may, with little study, be competent to keep the reckoning of a vessel. The Diagram of Navigation exhibits, *at one glance*, the whole system of calculation in making up the reckoning. From very little use it impresses clearly, and distinctly, on the mind, and fixes in the memory, all the necessary rules for performing the problems in Plane, Traverse, Parallel, Middle Latitude, and Mercator's Sailing, without the aid of any other book or table. The extreme facility of performing, by it, all problems in right angled plane trigonometry cannot fail to recommend it to students and teachers in Geometry, Navigation and Surveying, for the aid it will afford them in communicating, by sensible demonstration, the first principles in these sci-

ences. It will be, therefore, a valuable acquisition in all schools where these branches are taught.

In regard to the Diagram of Navigation it may not be improper for me to say, it is my own invention. That any similar scheme had been used, or projected, I had no knowledge till I exhibited, in 1820, a manuscript of the Diagram to a learned friend,* who first made me acquainted with the *Quartier de Réduction*, as described and explained by *Bézout*, in his *Traité de Navigation.*

The Diagram of Navigation, and the Quartier de Reduction are projected on the same geometrical principles. The *arrangement* of these principles for practical utility is however essentially different in the two instruments.

The *graduated* and appropriate *Index* of the *Diagram* relieves it from the troublesome *thread*, and the confused *concentric arcs* of the *Quartier de Réduction.* Beside, the annexation of the Arc of expanded Degrees of latitude renders the Diagram of Navigation the most convenient system in use for making up the reckoning. Those acquainted with the Quartier de Reduction will find its deficiences supplied by the Diagram. The Arc of the Diagram being graduated to tenths of a degree, and which, by a vernier scale, may be reduced to minutes, renders it very convenient and accurate in performing many common problems which are somewhat troublesome by the method of inspection, where the tables are only calculated to whole degrees. For instance, if the exact course and distance were required for 175 miles difference of latitude and 135 miles departure—the Diagram, from a

* The Hon. John Davis, LL. D. Judge of the U. S. District Court, in Massachusetts.

simple operation, gives the course 37° and $6\frac{1}{2}$ tenths or 37°39′, and the distance 221 miles; and these results are the same by logarithms. But the mean of *two* operations by inspection gives the course 37° 30′, and the distance 220.5.

The Diagram if carefully used is not liable to get out of order. If at any time it may be supposed to be in error, its accuracy may be tested many obvious ways; among others, set the index on 60° of latitude, and any number traced from the equator will be doubled on the index—according to the theorem that the secant of 60° is equal to twice the radius; from which is this rule, Rad.: mer. dist. :: sec. of lat.: diff. of lon. In publishing the remarks which accompany the Diagram, after the description and use, my original intention was to have added only the rules, examples and illustrations which are contained in pages 16 to 31;—But to make it more acceptable to those may who wish to become acquainted with the method of resolving the problems in Navigation arithmetically, I have added the second section, containing easy theorems from which the solutions are derived. The projection of the triangle given by the theorem, being readily formed by the Diagram, if it be well examined in connection with the reading of the solution, the learner will soon possess the idea of the proportions of the several terms. As all right-angled plane triangles may be, immediately, and accurately resolved by the diagram, it may be extensively useful for many valuable purposes. The examples given in this small work are necessarily few; but, by the Diagram, in the hands of a teacher they may be unlimited. In the execution of the work I have aimed at correctness; for any errors, which may have escaped my observation, I claim indulgence.

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ERRATA.—Page 13,art. 36, for *Circles of latitude are therefore*,read *Parallels of latitude are therefore circles.* 23,last line, for *is in* read *in is.* 31, last line but one of article 105, for *add.mer. lat.* read *add. mer. diff. lat.* 33, in fifth line of article 114, for 219 1·2 read 233 1·2. 34, in last line of article 116, for 66° 56' read 67° 14'. 45, in third line of article 192, for *circle one* read *one circle.* 56, in fifth line of article 230, erase the words *being its co-sine.*

DESCRIPTION AND USE

OF A

DIAGRAM OF NAVIGATION.

SECTION FIRST.

1. Navigation, in a special sense, is the art of sailing a vessel from one place to another.

2. That part of Navigation called *Nautical Astronomy* comprehends observations of the heavenly bodies, by suitable instruments, whereby the ship's place may be known with more certainty.

Description of the Diagram of Navigation.

3. The Diagram of Navigation is a geometrical canon of sines, tangents, and secants, corresponding to any given radius.

4. It is in the form of a Quadrant, or quarter circle, and exhibits a series of right-angled plane triangles, whose sides and angles are measured by marginal figures.

5. It has two sides perpendicular to each other, and a graduated Arc.

6. The sides are divided into equal parts; to which parts are drawn lines parallel to the sides; which lines,

intersecting at right angles, form the plane of the Diagram into equal squares.

7. The equal parts into which the sides are divided are numbered at every decimal division, and every fifth line being larger than those which are intermediate, any line may be readily traced to its margin, where its value is given.

8. The Arc is divided into eight parts, corresponding to the eight points of the quarter compass; which are numbered from left to right; and each point is subdivided into half and quarter points.

9. The Arc is also divided into ninety degrees; which are numbered at every division of five degrees, from left to right, and also from right to left; and each degree is subdivided into tenths.

10. The Arc may be divided into any given number of equal parts.

11. On the centre angle of the Diagram is fitted a moveable Index; one edge of which (that which coincides with the pivot on which it moves) is graduated to the equal parts of the sides.

12. From the centre angle to the arc are drawn rhumb lines for the points of the compass, as a guide to catch the eye readily on these points, and as shewing how the index forms the hypothenuse, or longest side of a right-angled triangle.

13. The divisions on the sides, and on the index may be valued as the figures express, or in any other convenient numbers resulting from multiplication or division, as in any scale of equal parts. Thus 100 may be used as $\frac{1}{10}$, as 10, as 100, as 1000, as 10,000, &c.—The subdivisions being always in a decimal ratio to the number assumed.

14. To the main arc of the diagram is annexed an arc of expanded degrees of latitude for the meridional parts used in Mercator's sailing. This arc is also divided into degrees and tenths, and is numbered at every division of five degrees, beginning at 10° and ending at 62°. The process of obtaining the meridional parts

by this arc is very simple, and will be explained under Mercator's sailing.

15. The left side of the diagram herewith published is marked MERIDIAN, and the right EQUATOR. The first column on the arc is marked COURSE IN POINTS, the second COURSE IN DEGREES, the third LATITUDE, and the fourth EXPANDED DEGREES OF LATITUDE. Their several uses will be hereinafter explained.

16. By a vernier fixed to the index the tenths of degrees may be subdivided into minutes. At each end of the main arc is an arc of excess.

17. For permanent use the diagram may be made on hard metal; but which would be too expensive for general circulation. I therefore publish it as an engraving, to render it more accessible to those for whose use it is intended.

Use of the Diagram of Navigation.

18. The Diagram of Navigation may be used to ascertain readily, and accurately the unknown parts of any right-angled plane triangle, if at least there be given one of the sides, and an acute angle opposite, or contiguous to the side given.

19. It is therefore very conveniently applicable to the solution of all problems in navigation relating to plane, traverse, parallel, middle latitude, and Mercator's sailing.

20. For this purpose the arc represents a quarter section of the HORIZON, one side the MERIDIAN, and the other the EQUATOR.

21. Either side may be assumed as meridian; but in practice it will be found most convenient to use one side constantly as meridian, and the other as equator. Throughout this work I shall use the left side as meridian, and the right as equator, and they are thus designated on the engraved diagram; the results will be applied to that quarter of the horizon which corresponds to the given problem.

22. By the description of the diagram it will be seen that it is an instrument, or table of geometrical ratio and proportion; by which any three numbers being given a fourth proportional is readily found. *Example.*—As 220 is to 160 so is 110 to 80. *Illustration.*—On either side of the diagram take 220, and trace its line till it intersects the line of 160 taken from the other side. At the point of intersection set the index; then 110 taken on the side which 220 was, and traced to the graduated edge of the index will intersect the line 80 from the side 160 was taken.

23. By fixing sights on the index the diagram becomes a convenient Theodolite to take the angles of objects on the land; and by fixing one of its sides perpendicular to the horizon the altitude of objects may be measured by it.

24. In all references to the INDEX, the graduated edge only is to be understood.

Remarks for using the Diagram.

25. The COURSE is taken on the ARC in points, or in degrees, counted from *left* to *right.*

26. The DISTANCE on the INDEX, in miles.

27. The DIFFERENCE OF LATITUDE, on the MERIDIAN, in miles.

28. The DEPARTURE, on the EQUATOR,* in miles.

29. The MERIDIAN DISTANCE on the EQUATOR, in miles.

30. The DIFFERENCE OF LONGITUDE, on the INDEX in miles, for *parallel and middle latitude sailing;* but for *Mercator's sailing*, on the EQUATOR.

31. The *meridional* DIFFERENCE OF LATITUDE, on the MERIDIAN, in *meridional parts.*

32. The LATITUDE, and the *middle* LATITUDE are counted on the arc marked LATITUDE, and from *right* to *left.*

* The departure being the sine of the course, is always parallel to the equator. If on any course the distance terminates at the equator, the departure is a section of the equator. If the distance terminates at a parallel of latitude, the departure is a section of that parallel; in either case the admeasurement on the equator is the same.

Definitions.

33. The earth is a spheroid, being a little compressed at the poles; but for all purposes of practical navigation it is considered a globe.

34. Every place or point on the earth, is designated by its latitude and its longitude.

35. *Latitude* commences at 0 on the equator, and is counted on a meridian, 90° north, to the north pole, and 90° south to the south pole.

36. *Circles of Latitude* are therefore parallel to the equator, and they decrease as the latitude approaches the pole.

37. *Longitude* commences at 0 on the *first* MERIDIAN, and is counted on the equator east or west 180°.

38. The *First Meridian* is the meridian of some chosen place, from which the longitude is reckoned.

39. Different nations have chosen different meridians for their first meridian. The English and the Americans reckon their longitude from the meridian of Greenwich, which is used as the first meridian in this work.

40. *Meridians* are arcs of semicircles subtended by the earth's axis. Their extremities are therefore in the poles of the equator. Two opposite meridians united form a great circle.

41. A ship's reckoning is kept in latitude and in longitude; and its object is to ascertain from it, at all times, the place of the ship on the ocean.

42. The elements of the reckoning are the course steered, and the distance sailed; from which are known the difference of latitude and the departure; and from the departure is deduced the difference of longitude.

43. If the course, and the distance could be exactly determined on all occasions, the aid of nautical astronomy would not be required to correct the reckoning.

44. But the course, and the distance are both subject to uncertainty from many causes.

45. The course is affected by the variation of the compass, lee-way, currents, heave of the sea, an unsteady helm, &c.

46. The distance is affected by the inaccurate measure of the ship's progress, or rate of sailing; and also by uncertain currents, and heave of the sea.

47. A certain reliance cannot therefore be had on the course or the distance; nevertheless after making such allowance for errors which may be known, and for such others as the judgment of the experienced mariner may dictate, the course, and the distance are usually assumed, and concluded on as true in all operations regarding the reckoning; which is corrected by nautical astronomy as the opportunities for observations of the heavenly bodies afford.

PLANE SAILING.

48. In plane sailing no regard is had to the spherical figure of the earth; which, for this purpose is considered an even extended plane; on which the meridians are supposed parallel, and forming with the parallels of latitude, equal squares.

49. In plane sailing four terms of proportion are used, viz. the *Course*, the *Distance*, the *Difference of Latitude*, and the *Departure*; any two of which being given the other two may be found.

50. The COURSE is the direction in which the ship sails toward the horizon. It is ascertained by the compass, and is reckoned in points, and quarter points, or in degrees and minutes.

51. The COURSE is counted from the north to the east 8 points or 90°, from the north to the west 8 points or 90°, from the south to the east 8 points or 90°, and from the south to the west 8 points or 90 degrees.

52. The DISTANCE is the extent the ship sails on a direct course; it is reckoned in miles, and is determined by an instrument called the LOG.

53. The DIFFERENCE OF LATITUDE is the north, or south distance made on a direct course, and is reckoned in miles.

54. The DEPARTURE is the east or west distance, made on a direct course, and is reckoned in miles.

55. By these definitions it is evident that if a ship sail due north, or south, the difference of latitude will be equal to the distance sailed, and there will be no departure.

56. If she sail due east or west, the departure will be equal to the distance sailed, and there will be no difference of latitude.

57. But if she sail on an oblique course, that is between any two cardinal points, she will make both difference of latitude and departure; and each in a ratio as she sails more toward the one than the other.

58. If a ship sail more toward the north, or south, than the east or west, the difference of latitude will exceed the departure.

59. And if she sail more toward the east or west, than the north or south, the departure will exceed the difference of latitude.

60. But in *no case* can the difference of latitude, or the departure exceed the distance sailed.

61. The analogy which plane sailing has to plane trigonometry is, that on all oblique courses the *distance*, the *difference of latitude*, and the *departure* make the three sides of a right-angled triangle; in which the distance always represents the hypothenuse or longest side,—the difference of latitude and the departure the two sides which comprise the right-angle.

62. And, as an angle in a semi-circle is a right-angle, so will the two sides made by the difference of latitude and the departure be exactly contained within the arc of a semi-circle whose subtense, or chord is equal to the distance sailed, on any oblique course.

63. In plane sailing there are six cases.

CASE 1.

64. *Given—the* COURSE *and* DISTANCE, *to find the* DIFFERENCE *of* LATITUDE *and* DEPARTURE.

Rule.—Set the index on the course;—then from the distance on the index, trace the nearest line to the meridian for the difference of latitude,—and the nearest line to the equator for the departure.

Example.—A ship sails north 3 points east, 243 miles,—required the difference of latitude and departure.

Here—the index set on the course 3 points, 243 the distance taken on the index, intersects the line to the meridian 202 for difference of latitude, and the line to the equator 135 for departure.

Example.—A ship sails S. 30 deg. W. 164 miles—required the difference of latitude and departure.

Here the index set on the course 30 deg. (counted from the meridian) 164 the distance gives the diff. of lat. 142 and the departure 82.

Remark.—By these examples it is seen that the distance, the difference of latitude, and the departure all meet at the same point of intersection on the rhumb-line of the course; and which shews that the point of intersection of any two given terms is also that of the two terms sought. By well considering this important fact, learners will soon acquire a knowledge of the relations of the general terms used in Navigation.

CASE 2.

65. *Given—the* COURSE *and* DIFFERENCE OF LATITUDE, *to find the* DISTANCE *and* DEPARTURE.

Rule.—Set the index on the course; then the diff. lat. taken on the meridian, and traced to the *graduated edge* of the index, will give the distance on the index, and the line to the equator for the departure.

Example.—A ship sails north $3\frac{1}{4}$ points east, and the diff. of lat. is 143 miles; required the distance and the departure.

Here the index set on the course $3\frac{1}{4}$ points, the diff. lat. 143 taken on the meridian, will give the distance 178 on the index, and the line to the equator 106 for the departure.

CASE 3.

66. *Given—the* COURSE *and* DEPARTURE, *to find the* DISTANCE *and* DIFFERENCE OF LATITUDE.

Rule.—Set the index on the course; then the departure taken on the equator will give the distance on the index, and the line to the meridian for the diff. of latitude.

Example.—A ship sails north $2\frac{1}{2}$ points east till her departure is 116 miles; required the distance and diff. of latitude.

Here the index set on the course $2\frac{1}{2}$ points, the departure 116 taken on the equator will give the distance 246 on the index, and the line 217 to the meridian for diff. of latitude.

CASE 4.

67. *Given—the* DISTANCE *and* DIFFERENCE OF LATITUDE *to find the* COURSE *and* DEPARTURE.

Rule.—Set the index to meet the diff. lat. from the meridian, with the distance on the index, it will be on the course, and the line to the equator for the departure will be given.

Example.—A ship sails southwesterly 212 miles, and her diff. of lat. is 187; required the course and departure.

Here the index set to meet the distance 212 with the diff. of lat. 187 will be on the course $2\frac{1}{2}$ points (or s. s. w. half w.) and will give the line to the equator 100 for the departure.

CASE 5.

68. *Given—the* DISTANCE *and* DEPARTURE, *to find the* COURSE *and* DIFFERENCE OF LATITUDE.

Rule.—Set the index to meet the departure with the distance, it will be on the course, and will give the line to the meridian for the diff. of lat.

Example.—A ship sails southwesterly 104 miles, and her departure is 64; required the course and diff. of latitude.

Here the index set to meet the departure 64, with the distance 104 will give the course 38 deg. (or s. 38° w.) and the diff. of lat. 82.

CASE 6.

69. *Given—the* DIFFERENCE OF LATITUDE *and* DEPARTURE, *to find the* COURSE *and* DISTANCE.

Rule.—Set the index to meet the difference of latitude with the departure, it will be on the course, and will give the distance on the index.

Example.—A ship sails northeasterly, and her diff. of lat. is 89 miles, and her departure 36; required the course and distance.

Here the index set to meet the diff. of lat. 89 with the dep. 36, it will be on the course 22 deg. (or N. 22 deg. E.) and will give the distance 96 miles.

TRAVERSE SAILING.

70. When a ship sails on several courses during a day, or any given period of time, it is called traverse sailing. These courses and the distances are entered in the traverse table so called; and on each separate course and distance is calculated, by case 1, plane sailing, the difference of latitude and the departure; which are entered in the table, in their proper columns. This done, the several columns are added up, and the

less latitudes taken from the greater, and the less departures from the greater. The remainders will shew the true difference of latitude and departure made by all the courses; and with this diff. of lat. and departure, the direct course and distance is found, by case 6, plane sailing. *Example.*—A ship sails the following courses and distances, viz: N. N. E. 13 miles, W. N. W. 13, N. E. 24, S. E. by E. half E. 17, S. by W. quarter W. 33; required the direct course and distance.

TRAVERSE TABLE.

COURSES.	DIS.	DIFF. LAT. N.	DIFF. LAT. S.	DEPARTURE. E.	DEPARTURE. W.
N. 2 pts. E.	13*	12 0	0 0	5 0	0 0
N. 6 pts. W.	13	5 0	0 0	0 0	12 0
N. 4 pts. E.	24	17 0	0 0	17 0	0 0
S. $5\frac{1}{2}$ pts. E.	17	0 0	8 0	15 0	0 0
S. $1\frac{1}{4}$ pts. W.	33	0 0	32 0	0 0	8 0
		34	40	37	20
			34	20	
			6	17	

Here the two remainders shew that the diff. lat. is 6 miles South, and the departure is 17 E.—which by case 6 in plane sailing give the direct course S. 70 deg. 34 m. E. and the distance 18 miles.

PARALLEL SAILING.

71. In parallel sailing the earth is considered a globe, on which the meridians all meet at the poles.

* When the numbers given are too small for convenient operation, substitute larger ones. Thus for 13 take 130 and the subdivisions will be tenths. The operations should always be made as far from the centre angle as may be convenient.

72. The distance between any two meridians continually decreases in progressing from the equator to the pole, where it is nothing.

73. In parallel sailing a ship always sails due East or West.

74. It has three cases; and in which these three terms are used, viz: the PARALLEL OF LATITUDE, the MERIDIAN DISTANCE (being the distance sailed,) and the DIFFERENCE OF LONGITUDE, any two of which may be given to find the other.

CASE 1.

75. *Given—the* DIFFERENCE OF LONGITUDE *between two places in the same* PARALLEL OF LATITUDE *to find the* MERIDIAN DISTANCE.

Rule.—Set the index on the given parallel,—then from the difference of longitude, taken on the index, trace the nearest line to the equator for the meridian distance.

Example.—The difference of longitude between two places in the parallel of 45 deg. of latitude is 140 miles—required the meridian distance.

Here the index set on the given parallel 45 deg. the diff. of longitude 140 taken on the index gives the line to the equator 99 the meridian distance.

76. By this case and rule is found the number of miles which make a degree of longitude in any latitude, by setting the index on the given latitude,—then from 60 on the index, (being the miles of a degree of longitude on the equator) trace the nearest line to the equator for the miles required.

Example.—How many miles make a degree of longitude in latitude 40 deg.?

Here set the index on 40 deg. counting from the equator,—then from 60 on the index the line to the equator is 46, the miles required.

CASE 2.

77. *Given the* MERIDIAN DISTANCE *between two places in the same* PARALLEL OF LATITUDE, *to find the* DIFFERENCE OF LONGITUDE.

Rule.—Set the index on the given parallel,—and the meridian distance taken on the equator, will give the difference of longitude on the index.

Example.—The meridian distance between two places in the parallel of 36 deg. of lat. is 110 miles—required the difference of longitude.

Here the index set on 36 deg. of lat. 110 on the equator gives on the index 136 the difference of longitude.

A ship in lat. 48 deg. sails west 150 miles, what is her difference of longitude? *Ans.* 224 miles.

A ship in lat. 32 deg. sails east 117 miles, what is her difference of longitude? *Ans.* 138 miles.

A ship in lat. 10 deg. sails west 60 miles, what is her difference of longitude? *Ans.* 61 miles.

A ship on the equator sails east 60 miles, what is her difference of longitude? *Ans.* 60 miles.

CASE 3.

78. *Given—the* DIFFERENCE *of* LONGITUDE *and the* MERIDIAN DISTANCE, *to find the* PARALLEL *of* LATITUDE.

Rule.—Set the index to meet the difference of longitude with the meridian distance, it will be on the parallel of latitude.

Example.—A ship sails east 105 miles, and her diff. of lon. is 120—required the latitude sailed in.

Here the diff. of lon. 120 with the meridian distance 105 give the latitude 29 deg.

A ship sails west 115 miles, and her diff. of lon. is 146, what is the latitude? *Ans.* 38 deg.

A ship sails east 125 miles, and her diff. of lon. is 203, what is the latitude? *Ans.* 52 deg.

MIDDLE LATITUDE SAILING.

79. In middle latitude sailing the earth is considered a globe, as in parallel sailing.

80. It is compounded of plane and of parallel sailing, the middle latitude being substituted as an artificial parallel, by which the departure, found by the rules in plane sailing, is reduced to longitude by the rules in parallel sailing.

81. The middle latitude is the half sum of the two latitudes, when they are both on the same side of the equator; but when the latitudes are on each side of the equator, it is half the difference of the two latitudes.

82. There are seven cases; and in which these six terms are used, viz: course, distance, difference of latitude, departure, difference of longitude, and middle latitude.

CASE I.

83. *Given—the* LATITUDES *and the* LONGITUDES *of two places to find the* COURSE *and* DISTANCE.

Rule.—Set the index on the middle latitude; and from the difference of longitude, taken on the index trace the nearest line to the equator for the departure; then the difference of latitude, and the departure will give the course and distance, as in case 6, plane sailing.

Example.—What is the course and distance from Cape Cod Light-house in lat. 42 deg. 05 N. lon. 70 d. 04 W. and Mt. Desert Rock in lat. 43 deg. 52 N. and lon. 68 d. 09 W.?

Lat.	43 52	Lon.	70 04	Diff. lat. 107 N.
	42 05		68 09	
Sum	85 57		1 55—diff. of lon.=115 E.	

$\frac{1}{2}$ is 42 58$\frac{1}{2}$ mid. lat.

Here the index set on the middle lat. 42 deg. $58\frac{1}{2}$ (or 43 deg.) the diff. lon. 115 on the index gives the departure 84 on the equator; then the diff. lat. 107 and the departure 84, give the course $38\frac{2}{10}$ deg. or N. 38 deg. 12 m. E. and the distance 136 miles.

CASE 2.

84. *Given—both* LATITUDES *and the* DEPARTURE, *to find the* COURSE, DISTANCE, *and* DIFFERENCE OF LONGITUDE.

Rule.—Set the index to meet the difference of latitude with the departure, it will give the course and distance; then set the index on the middle latitude, and the departure on the equator will give the difference of longitude on the index.

Example.—A ship in lat. 42 deg. N. sails northeasterly till her diff. of lat. is 180 miles, and her departure 104; required the course, distance, and the difference of longitude.

Here the diff. lat. 180 and the dep. 104 give the course N. 30 deg. E. and the distance 208 miles; then the middle lat. 43 deg. 30 m. and the departure 104 give the diff. of lon. $143\frac{1}{2}$ miles.

CASE 3.

85. *Given—one* LATITUDE, *the* COURSE *and the* DISTANCE, *to find the* DIFF. OF LAT. *the* DEPARTURE, *and the* DIFF. OF LONGITUDE.

Rule.—The course and distance give the diff. of lat. and the dep. as by case 1 plane sailing; then the middle lat. and the dep. give the diff. of lon.

Example.—A ship in lat. 34 deg. N. sails N. $3\frac{1}{4}$ points W. 193 miles; required the diff. of lat. the departure, and the diff. of lon.

Here the course $3\frac{1}{4}$ points and the distance 193 give the diff. lat. 155, and the departure 115; hence the lat. is in 36 deg. 35 m. N. and the mid. lat. is 35 deg. 18,

which with the departure 115 give the diff. of lon. 141 miles.

CASE 4.

86. *Given—both* LATITUDES *and the* COURSE, *to find the* DISTANCE, *the* DEPARTURE, *and the* DIFF. OF LON.

Rule.—Set the index on the course, and the diff. lat. will give the distance and the departure; then set the index on the middle latitude, and the departure will give the diff. of lon.

Example.—A ship in lat. 44 deg. N. sails N. 5 points W. till she is in lat. 46 deg. N.; required the distance, the departure, and the diff. of lon.

Here the course 5 points and the diff. lat. 120 give the distance 216 miles, and the departure 179½; then the middle lat. 45 deg. and the dep. 179½ give the difference of lon. 254 miles.

CASE 5.

87. *Given—both* LATITUDES *and the* DISTANCE *to find the* COURSE, DEPARTURE, *and* DIFF. OF LONGITUDE.

Rule.—Set the index to meet the diff. of lat. with the distance, it will give the course and departure; and the middle latitude with the departure will give the difference of longitude.

Example.—A ship in lat. 28 deg. N. sails northeasterly 217 miles, and is in lat. 31 deg. 04 m. N. required the course, departure, and difference of longitude.

Here the distance 217 with the diff. lat. 184 give the course N. 32° E. and the departure 115; which with the middle lat. 29 deg. 32 m. give the diff. of longitude 132 miles.

CASE 6.

88. *Given—one* LATITUDE *the* COURSE *and* DEPARTURE, *to find the* DISTANCE, DIFF. LAT. *and* DIFF. LON.

Rule.—Set the index on the course, and the departure will give the distance and diff. of lat.; then the departure and middle latitude will give the diff. of lon.

Example.—A ship in lat. 26 deg. N. sails N. 5 points W. till her departure is 172 miles; required the distance, the diff. of lat. and the diff. of lon.

Here the course 5 points and the departure 172 give the distance 207, and diff. of lat. 115; and the departure 172 with the middle lat. 26 deg. 58 m. (or 27 deg.) give the diff. of lon. 193.

CASE 7.

89. *Given—one* LAT. DISTANCE *and* DEPARTURE, *to find the* COURSE, DIFF. LAT. *and* DIFF. OF LON.

Rule.—The distance and the departure will give the course, and diff. lat.; and the departure and middle lat. will give the diff. lon.

Example.—A ship in lat. 42 deg. N. sails southeasterly 119 miles, and her departure is 54; required the course, diff. lat. and diff. lon.

Here the distance 119 with the departure 54 give the course S. 27 deg. E. and the diff. lat. 106; and the middle lat. 41 deg. 07 m. with the departure 54 give the diff. lon. 72 miles.

MERCATOR'S SAILING.

90. In Mercator's sailing the earth is considered as an extended plane, *infinite* in length, and in breadth equal to the earth's circumference at the equator.

91. This projection was invented in 1566 by Gerard Mercator; its object is to represent on a plane the

true bearing or course from one place to another as it is on a globe.

92. In this projection the meridians are represented by straight parallel lines; by which the degrees of longitude in high latitudes are artificially expanded, being the same in all latitudes as on the equator. This expansion is in the ratio of radius to the cosine of the latitude.

93. In order therefore to represent the true course between two places, the degrees of latitude are also artificially expanded, and in the ratio of the secant of the latitude to radius.

94. The parts contained in the expanded degrees of latitude are called meridional parts; and the meridional difference of latitude is in the same ratio to the difference of longitude, that the proper difference of latitude is to the departure.

95. To find the meridional parts by the diagram, the index must be set to the given latitude on the arc of *expanded* degrees, and it will cut on the arc of latitude, the degrees, which reduced to miles give the required meridional parts.

Example.—Required the meridional parts for 33 deg. of latitude. The index set to 33 deg. on the expanded arc cuts 35 deg. on the arc of latitude, which reduced to miles give 2100 the meridional parts for 33 deg.

96. When whole degrees are cut on the arc of latitude, annex a cipher and multiply by 6; and when degrees and tenths are cut, annex the tenths *without* a point of separation, and multiply by 6. Thus,

Deg.	*m.*		*deg.*	*tenths.*						*Mer. parts.*
33	00	cuts	35	0	used as	350	mult.	6	equal	2100
29	36	"	31	0	"	310	"	6	"	1860
26	30	"	27	5	"	275	"	6	"	1650
44	00	"	49	1	"	491	"	6	"	2946
50	00	"	57	9	"	579	"	6	"	3474
56	00	"	67	9	"	679	"	6	"	4074

97. In Mercator's sailing these six terms are used, viz :—*Course, Distance, Departure, Proper difference of Latitude, Meridional difference of Latitude,* and *Difference of Longitude.*

CASE 1.

98. *Given—the* LATITUDES *and the* LONGITUDES *of two places, to find the* DIRECT COURSE *and* DISTANCE.

Rule.—Set the index to meet the meridional difference of latitude counted on the meridian, with the difference of longitude, counted on the equator; the index will be on the course; then, the index remaining on the course, the proper difference of latitude, traced from the meridian, will give the distance on the index.

Example.—What is the course and distance from Belfast, in lat. 44° 27′ N. and lon. 69° 00′ W. and Quebec, in lat. 46° 48′ N. and lon. 71° 05′ W.? This case prepared stands thus, diff. lat. 141 miles N. mer. diff. lat. 202, diff. lon. 125 W.

Then the index set to meet the mer. diff. lat. 202 with the diff. lon. 125, it will be on the course 31° and 7½ tenths, which will be north 31° 45′ west; then the index remaining on the course, the proper diff. of lat. 141 gives the distance 166 geographical miles.

Example 2d.—What is the course and distance from Castine, in lat. 44° 24′ N. lon. 68° 46′ W. and Plymouth Lights in lat. 41° 59′ N. and lon. 70° 34′ W.?

Answer. Course S. 28° 29′ W.
Distance 165 miles.

CASE 2.

99. *Given—both* LATITUDES *and the* DEPARTURE, *to find the* COURSE, DISTANCE, *and* DIFF. OF LON.

Rule.—Set the index to meet the proper difference of latitude with the departure, it will give the course and distance; then the index remaining on the course, the meridional difference of latitude will give the difference of longitude.

Example.—A ship in lat. 42 deg. 05 m. N. sails northeasterly till she is in lat. 43 deg. 52 m. N. and the departure is 84 miles; required the course, distance, and diff. longitude. This case prepared stands thus, the proper diff. lat. 107, dep. 84, and mer. diff. lat. 147.

Then the proper diff. lat. 107 with the departure 84, give the course N. 38 deg. 08 m. E. and the distance 136; and the mer. diff. lat. 147 with the course, give the difference of longitude 115 miles.

CASE 3.

100. *Given—one* LATITUDE, *the* COURSE, *and* DISTANCE, *to find the* DIFF. OF LAT. *and* DIFF. OF. LON.

Rule.—Set the index on the course and the distance will give the diff. lat. by which the other latitude may be known; then the index remaining on the course, the meridional difference of latitude, will give the difference of longitude.

Example.—A ship in lat. 40 deg. N. sails N. 42 deg. E. 187 miles; required the diff. lat. and the diff. lon.

Here the course 42 deg. and the distance 187 give the diff. lat. 139, which added to the lat. left gives 42 deg. 19 m. the lat. in—Hence the mer. diff. lat. is 184, which with the course 42 deg. gives the diff. lon. 166 miles.

CASE 4.

101. *Given—both* LATITUDES *and the* COURSE *to find the* DISTANCE *and the* DIFF. OF LON.

Rule.—Set the index on the course, and the diff. lat. will give the distance; then the index remaining on the course the mer. diff. of lat. will give the diff. of lon.

Example.—A ship in lat. 44 deg. sails S. 3 points W. till she is in lat. 41 deg. 22 m. N.; required the distance, and the diff. of lon. This case prepared stands, diff. lat. 158, mer. diff. lat. 215.

Then the course 3 points, with the diff. of lat. 158 give the distance 190, and the mer. diff. lat. 215 with the course 3 points give the diff. of lon. 144 miles.

CASE 5.

102. *Given—both* LATITUDES *and the* DISTANCE, *to find the* COURSE *and* DIFFERENCE OF LONGITUDE.

Rule.—Set the index to meet the distance with the difference of latitude,—it will be on the course;—and the index remaining on the course, the meridional difference of latititude will give the difference of longitude.

Example.—A ship in lat. 34 deg. N. sails northwesterly 220 miles, and is in lat. 36 deg. 51 m. N. required the course and the difference of longitude.

Here the distance 220 with the diff. lat. 171 give the course N. 39 deg. W. which with the mer. diff. lat. 210 give the diff. of longitude 170 miles.

CASE 6.

103. *Given—one* LATITUDE, *the* COURSE *and* DEPARTURE, *to find the* DISTANCE, DIFFERENCE OF LATITUDE *and* DIFFERENCE OF LONGITUDE.

Rule.—Set the index on the course, and the departure will give the distance and the difference of latitude; and the index remaining on the course, the meridional difference of latitude will give the difference of longitude.

Example.—A ship in latitude 24 deg. S. sails S. $2\frac{1}{2}$ points W. till her departure is 116 miles; required the distance, the diff. of latitude, and the diff. of longitude.

Here the course $2\frac{1}{2}$ points, and the departure 116 give the distance 246, and the diff. of latitude 217—hence the lat. in is 27 deg. 37 m. and the mer. diff. of lat. is 241, which with the course $2\frac{1}{2}$ points give the diff. lon. 129 miles.

CASE 7.

104. *Given—one* LATITUDE, *the* DISTANCE *and* DEPARTURE, *to find the* COURSE, DIFFERENCE OF LATITUDE *and* DIFFERENCE OF LONGITUDE.

Rule.—Set the index to meet the distance with the departure, it will give the course, and difference of latitude; and the course and mer. diff. of latitude will give the difference of longitude.

Example.—A ship in lat. 16 deg. south sails northwesterly 297 miles, and her departure is 210; required the course, the diff. of lat. and the diff. of longitude.

Here the distance 297 with the departure 210 give the course N. 4 pts. W. and the diff. of latitude 210; hence the latitude in is 12 deg. 30 m. S. and the mer. difference of latitude is 217, which with the course 4

points give the diff. of longitude also 217; and which proves what is said in 94, *that the mer. diff. of latitude is to the diff. of longitude as the proper diff. of latitude is to the departure.*

CASE 8.

105. *Given—one* LATITUDE, *the* COURSE *and the* DIFFERENCE OF LONGITUDE, *to find the* DISTANCE *and the* DIFF. OF LATITUDE.

Rule.—Set the index on the course, and the difference of longitude will give the meridional difference of latitude, by which the proper difference of latitude may be known, and which with the course will give the distance.

Example.—A ship in lat. 42 deg. N. sails N. 38 deg. E. and her diff. of longitude is 125 miles; required the distance, and diff. of latitude.

Here the course 38 deg. with the diff. lon. 125 give the mer. diff. of lat. 160; hence the proper diff. of lat. is 117, which with the course gives the distance 149 miles.

Lat. left 42 deg. equal 2782 mer. parts.
Add. mer. lat. 160

2942=43° 57′ the lat. in.

106. To find by the diagram the value of any given meridional parts in degrees of lat.—divide them by 60, which reduces them to degrees and minutes; then set the index on the arc of latitude, to the degrees found, and it will cut on the expanded arc, the true degrees of latitude answering to the meridional parts given.

Example.—What is the latitude for 2100 meridional parts? Here 2100 divided by 60 gives 35 deg. to which set the index on the proper arc of latitude, and it will cut on the expanded arc 33 deg. being the latitude for 2100 meridional parts. It is seen that this

operation is the reverse of that by which the meridional parts are found for any given latitude, as shewn in No. 95, page 26.

CORRECTING THE DEAD RECKONING.

107. If the courses and distances which a vessel is supposed to sail were uniformly true, the difference of latitude by account would always agree with that from an observation of the heavenly bodies. But during a voyage it happens frequently that the latitude in by account differs from that by observation.

108. This difference arrises from an error in the course, or in the distance, and often in both.

109. When the course is more toward the meridian than the equator, the difference in the latitudes it is supposed arises from an error in the distance, rather than in the course.

110. But if the course is more toward the equator than the meridian, it is presumed the difference in the latitude arises from an error in the course rather than in the distance.

111. Various methods have been proposed to rectify these errors; but the most eminent mathematicians agree in this conclusion,—that, if after a careful re-examination of the incidents which affect the course and distance, there should still exist a difference between the latitude by account, and that by observation, the course and the distance should be corrected by the difference of latitude from observation, with the departure from account—leaving the departure by account unaltered.

112. *Example.*—A ship in lat. 44 deg. N. and lon. 60 deg. W. is supposed to sail N. 22 deg. 30 m. E. (N. N. E.) 264 miles, by which her diff. lat. would be 244 and her departure 101—and she would thereby be in lat. 48 deg. 04 m. N. and in lon. 57 deg. 35 m. W.; but by an observation of the sun, the ship is found to

be in lat. 47 deg. 40 m. N. what correction must be made to the course and distance? Here the diff. lat. by observation is 220, which with the departure 101, give the corrected course N. 24 deg. 39 m. E. and the distance 242 miles. As the mid. lat. is not materially changed, the longitude remains the same.

113. *Example.*—A ship in lat. 44 deg. N. and lon. 60 deg. W. is supposed to sail N. 67½ deg. E. (E.N.E.) 264 miles, by which her diff. of lat. would be 101 and her departure 244—and thereby she would be in lat. 45 deg. 51 m. N. and lon. 56 deg. 10 m. W.; but by an observation of the sun the ship is found to be in lat. 46 deg. 15 m. N.; required the correction to be made to the course and distance. Here the diff. of lat. by observation is 135 miles, which with the departure 244 give the corrected course 61 deg. 03 m. and the distance 279; and the middle lat. not being much altered, the longitude remains the same.

QUESTIONS FOR PRACTICE.

114. What is the course and distance from Nantucket light-house in lat 41 deg. 22 m. N. lon. 70 deg. 00 m. W. and Cape Sable in lat. 43 deg. 26 m. N. and lon. 65 deg. 32. W.? *Ans.* course N. 57° 55′ E.
distance 219½ miles.

For the method of operation see Rule in Case 1, Middle Latitude Sailing, No. 83; and Case 1st, Mercator's Sailing No. 98.

115. A ship from Portland in lat. 43 deg. 39 min. N. lon. 70 deg. 13 min. W. sails S. E. 256 miles; what latitude and longitude is she in?

Ans. in latitude 40° 38′ N.
long. 66° 09′ W.

See No. 85, and No. 100.

116. A ship leaves Boon Island in lat. 43 deg. 6 m. N. and lon. 70 deg. 31 m. W. and sails E. S. E. till her departure is 145 miles; what distance has she sailed, and what latitude and longitude is she in?

Ans. dist. sailed 157 miles.
lat. in 42° 06′ N.
lon. in 66 56 W.

See 88 and 103.

117. A ship leaving Boston light house in lat. 42 deg. 20 m. N. and lon. 70 deg. 54 m. W. sails E. by S $\frac{1}{2}$ S. and by observation it is found she is in lat. 40 deg. 59 m. N. what distance has she sailed; and what longitude is she in? *Ans.* distance sailed 279 miles.
lon. in 64° 57′ W.

See No. 86 and 101.

118. A ship from Isle of Holt in lat. 44 deg. north and lon. 68 deg. 31 m. W. sails southeasterly 224 miles and is in lat. 41 deg. 00 m. N. what course has she steered, and what lon. is she in?

Ans. course steered S. E. $\frac{3}{4}$ S.
lon. in 65° 29′ W.

See No. 87 and 102.

119. A ship bound to Philadelphia is in lat. 40 deg. 00 min. N. and in lon. 69 deg. W. what is her course and distance to Cape May in lat. 38 deg. 57 min. N. and lon. 74 deg. 57′ W.?

Ans. course S. 77° 04′ W.
distance 281$\frac{1}{2}$ miles.

120. A ship bound to Boston, and coming into the Bay, discovers Mount Desert Rock, which bears W.N.W. the true course, and distant 18 miles; Mount Desert Rock is in lat. 43 deg. 52 min. N. and lon. 68 deg. 09′ W. what lat. and lon. is the ship in, and what is her course and distance to Boston light-house which is in lat. 42 deg. 20 min. N. and lon. 70 deg. 54 min. W.?

Ans. the ship is in lat. 43° 45′ N. long. 67° 45′ W.
the course is S. 58° 28′ W. distance 162$\frac{1}{2}$ miles.

121. Two ships leave the same port at the same time; one sails East at the rate of six miles per hour, the other sails E. N. E. Ten hours after sailing the latter bears from the former due north. How far are the ships apart, and what distance has each sailed.

Set the index on 6 points (E. N. E.) and the ship sailing east having run 60 miles in 10 hours; therefore 60 on the side marked equator will give on the index 65, which is the distance the other ship has sailed, from which the nearest line to the side marked meridian gives 25 miles the distance between the ships.

122. Two ships in lat. 44 deg. 30 m. north are 216 miles apart; they both sail due south at equal rates. When they are in lat. 32 deg. 18 m. N. what will be the distance between them?

Set the index on lat. 44 deg. 30 m. and 216 on the side marked equator will give on the index 303; then set the index on the other lat. 32 deg. 18 m. and 303 on the index will give on the side marked equator 256 miles, being the distance between the two ships in lat. 32 deg. 18 m. N.

CURRENTS.

123. A current is a stream in the ocean, by which all bodies on its surface, within the limits of the current, are drifted toward that part of the horizon to which the current sets.

124. When a ship is affected by a current, the direction, or *set* of the current is entered in the traverse table as a course sailed; and the rate, or *drift* of the current is made up as a distance sailed.

125. On such course and distance the diff. lat. and departure are calculated, as on a course and distance sailed. *Example.*—A ship sailing N. E. at the rate of 6 miles per hour for 10 hours, it is ascertained that a

current, during that time, has set S. E. at the rate of 2 miles per hour. What is the direct course, and distance sailed for the 10 hours?

TRAVERSE TABLE.

Course.	*Distance.*	*Diff.* N.	*Lat.* S.	*Departure.* E.	W.
N. E.	60	42.4	0.0	42.4	0
S. E.	20	0.0	14.1	14.1	0
		42.4		56.5 E.	
	less	14.1			
		28.3 N.			

Here by the traverse table the diff. lat. is 28. 3 N. and the departure is 56. 5 E. which by case 6, plane sailing, give the direct course N. 63 deg. 24 m. E. and the distance 63 miles.

126. A Table of the degrees and minutes corresponding to the points and quarter points of the compass; being the angle which said points form with the meridian.

Points.	0	¼	½	¾
0	. . . .	2° 49′	5° 37′	8° 26′
1	11° 15′	14 04	16 52	19 41
2	22 30	25 19	28 07	30 56
3	33 45	36 34	39 22	42 11
4	45 00	47 49	50 37	53 26
5	56 15	59 04	61 52	64 41
6	67 30	70 19	73 07	75 56
7	78 45	81 34	84 22	87 11
8	90 00	. . . .	. . . .	. . . .

Determining the Latitude by Observation.

127. The latitude of a place may be determined by an observation of the altitude of the sun, moon or stars.

128. The most convenient and common method of determining the latitude at sea is by taking, with a quadrant, the meridian altitude of the sun.

129. The apparent altitude of the sun, and also that of the moon, requires four corrections, viz :—for semidiameter, dip of the horizon, parallax, and refraction.

130. To the apparent meridian altitude of the sun's lower limb, taken by a fore-obsersation from the deck of a common vessel, it is usual to *add* 12 for the correction for the semidiameter, dip, and parallax ; then the refraction being *subtracted*, the remainder is the true altitude, and which being taken from 90 deg. will give the sun's zenith distance.

131. If the sun bears south, the zenith distance is called north—if it bear north, it is called south.

132. If the sun's declination be of the same name as the zenith distance, *add* them together, and their *sum* is the latitude of the place of observation; but if they be of different names, that is one north and the other south, take the less from the greater, and their *difference* is the latitude, and of the same name with the greater.

133. The sun's declination is given in the nautical Almanac for every day at noon, at Greenwich. If the longitude of the ship be great from Greenwich, the declination must be corrected for the longitude the ship is in. This correction is too apt to be neglected by some navigators, and the neglect in some cases may occasion an error of some miles in the latitude observed.

SECTION SECOND.

134. The preceding section contains all the necessary rules and examples to enable a person of the most common capacity to keep the reckoning of a vessel by the diagram.

In this second section I propose to give some geometrical definitions, to shew the application of the diagram of navigation to right-angled plane trigonometry; to which will be added some easy theorems, from which are derived the solutions for calculating the usual problems of navigation by logarithms, which will be illustrated by examples; and the usual method of making up the reckoning by inspection from the calculated tables will be shewn.

Geometrical Definitions.

135. Geometry treats of magnitude or extension, in regard to its three dimensions *length*, *breadth*, and *depth* or *thickness.*

136. Extension in length only is represented by a *line.*

137. Extension in length and breadth is represented by a *surface.*

138. Extension in length, breadth and depth is represented by a *solid*, commonly called a *body.*

139. There cannot properly be any extension without the three dimensions, length, breadth and depth; nevertheless length may be considered without regard to breadth and depth, and length and breadth together may be considered without regard to depth.

140. Therefore a POINT may be considered without regard to length, breadth or depth; and it represents any assignable place or position, as the extremities, and the divisions of lines.

141. That part of geometry which treats of extension in length is called *longimetry;* that part which treats of length and breadth, or surfaces, is called *planimetry;* and that part which treats of solid bodies is called *stereometry.*

142. There are two sorts of lines, *right* and *curve.* A right, or straight line is the nearest distance between two points; and but one straight line can be drawn between two points.

143. A curve line continually changes its direction, and an infinite number of curve lines may be drawn between two points.

144. Lines to each other are *parallel, oblique, perpendicular* and *tangential.*

145. *Parallel lines* are in the same direction, and being infinitely produced would never meet.

146. *Oblique lines* are in different directions, and they meet when produced on the side of their nearest distance.

147. *Perpendicular lines* meet without inclining more to one side than to the other.

148. A *right* line is tangential to a *curve* line, when the right line meets and touches the curve line, but if produced would not cross it.

149. When two lines meet at the same point, the space comprised between them is called an *angle;* and the point of intersection is called the *angular point,* or the *summit,* or *vertex* of the angle; and it always represents the centre of a circle.

150. The direction of the lines give the value of the angle, which is measured on the arc of a circle described about the angular point.

151. If two right lines cross each other in any direction, four angles are formed, which together are equal to 360°, and the angles which are opposite are equal.

152. *Figure* in general is a space enclosed by lines upon all sides.

153. The least number of right lines which can include a space are *three.*

154. A figure of three sides is called a *triangle.*

155. A figure of four sides is called a *quadrilateral.*

146. Figures of more than four sides are in general called *polygons.*

157. If three points be placed in any position, not in a straight line, and these points be connected by straight lines, a plane triangle is formed.

158. A CIRCLE is a plane round figure, bounded by a uniform curve line.

159. Geometers also consider a circle as a polygon, whose sides are infinite.

160. From definitions 153, 154 and 159 it follows that the triangle, and the circle are *extreme* figures; the one having the *least* number of sides possible, and the other the *greatest* possible number. By which it would seem that their properties and their uses would be very remote; which however is not the case; for the properties of the one can hardly be shewn but by those of the other.*

161. The CIRCUMFERENCE of a circle is the uniform curve line which encloses the space called the circle. The circumference is also called the circle.

162. The CENTRE of a circle is a point within the circle, equally distant from all parts of the circumference.

163. The circumference may be divided into any number of equal parts. For the purposes of astronomy, navigation and surveying, geometers divide the circle into 360 equal parts called degrees—each degree is subdivided into 60 minutes—each minute into 60 seconds—each second into 60 thirds, and so on.

164. The RADIUS of a circle is a right line drawn from the centre to the circumference; and is half the diameter.

165. The DIAMETER of a circle is a right line drawn from one point in the circumference to the opposite

* Deparcieux.

point through the centre, and is equal to twice the radius.

166. An ARC is any portion of the circumference, and is the measure of the angle comprised by two radii, drawn from the centre, one to each extremity of the arc.

167. The COMPLEMENT of an arc, or angle is what the arc or angle wants of a quadrant, or quarter circle, and it is known by subtracting the arc or angle from 90 deg.

168. The SUPPLEMENT of an arc or angle is what the arc or angle wants of a semicircle, and is known by subtracting the arc or angle from 180 degrees.

169. The CHORD of an arc is a right line drawn from one extremity of the arc to the other.

170. The SINE of an arc or angle is a right line drawn from one extremity of the arc, perpendicularly upon the radius, or diameter which is drawn to the other extremity of the same arc.

171. The CO-SINE of an arc or angle is the sine of the complement of that arc or angle—thus the *co-sine* of 29 degrees is the *sine* of 61 degrees. The co-sine is equal to that part of the radius contained between the angular point and the foot of the sine.

172. The VERSED SINE of an arc or angle is that part of the radius which is contained between the foot of the sine and the extremtiy of the arc. Therefore the *cosine* and *versed sine* are equal to radius.

173. The TANGENT of an arc or angle is a right line drawn perpendicularly from the extremity of the radius, which meets one extremity of the arc. The tangent is terminated by the secant of the same arc or angle.

174. The SECANT of an arc or angle is a right line drawn from the centre through one extremity of the arc, till it meets the tangent of the same arc or angle drawn from the other extremity of the same arc or angle. By the two last definitions it is seen that the tangent and the secant terminate each other.

175. The CO-TANGENT of an arc or angle is the tangent of the complement of that arc or angle.

176. The CO-SECANT of an arc or angle is the secant of the complement of that arc or angle.

177. A SEMICIRCLE is half a circle or 180 deg.

178. A QUADRANT is a quarter circle or 90 deg.

179. A SEXTANT is the sixth part of a circle or 60 deg.

180. An OCTANT is the eighth part of a circle or 45 deg.

181. A SEGMENT is any part of the circle bounded by an arc and its chord.

182. A SECTOR is an arc bounded by two radii.

183. An ANGLE is the space comprised by the meeting of two lines which are not in the same direction.—See No. 149.

184. All angles are *right* or *oblique.*

185. A RIGHT ANGLE is formed by two lines perpendicular to each other, and contains 90 deg.

186. An OBLIQUE ANGLE is greater, or less than a right angle or 90 deg.—if *greater* it is called *obtuse*—if *less* it is called *acute.*

187. As great use is made of the *chords, sines, tangents* and *secants,* it may aid the learner to refer to the diagram to ascertain the positions of these important properties of the circle.

188. On the diagram of navigation at the division of 100 as a radius, is a fine dotted curve line, which represents the arc or angle of 90 deg. If a line be drawn within this arc from one end of it to the other, that line will be a chord to the arc of 90 deg. and a line drawn from one end of this arc to any point of it will be a chord to that section of it which it subtends.—On the side marked meridian, and between the divisions 87 and 88 is a fine dotted straight line, which extends to the dotted curve line, and intersects it at the angle of 29 deg. to which set the index; the straight dotted line represents the *sine* of 29 deg. which being parallel to the side marked equator, its value is there found to be $48\frac{1}{2}$, which is the length or ratio of the sine of 29 deg. to the radius of 100. The other dotted straight line which leads from the same point of

intersection at the angle 29 deg. and to the side marked equator represents the *co-sine* of 29 deg. which being parallel to the side marked meridian its value is there given $87\frac{1}{2}$, which is the length, or ratio of the co-sine of 29 deg. to the radius of 100. If the length of the radius be increased, or diminished, the length of the sine and the co-sine will in like ratio be increased or diminished ; for if the radius be 200 on the index, the sine of 29 deg. will be 97, and the co-sine will be 175. In this example the side marked meridian is used as base. Now take the side marked equator as base, and count the degrees from right to left ; the index will be on 61 deg.—and it is seen that the co-sine of 29 deg. becomes the sine of 61 deg. and the sine of 29 deg. becomes the co-sine of 61 deg.

189. The line drawn perpendicular from the radius 100, at one end of the arc of 29 deg. is its tangent, which is terminated by the secant drawn through the other end of the same arc, which secant is represented by the graduated edge of the index, and its value is found on the index, if set on 29 deg. to be 114 nearly ; and the tangent being parallel to the side marked equator, is there found to be $55\frac{1}{2}$ nearly.

190. As the angle increases, the sine, tangent, and secant will also increase ; but the co-sine, co-tangent and co-secant will decrease. *Example.*—Set the graduated edge of the index to the side marked equator on 0. Let the radius be 100—of 0 the sine and the tangent will be negative, the co-sine and secant will be equal to the radius, and the co-tangent and the co-secant will be infinite. Now set the index on 8 deg. the sine will be 14 nearly, the tangent 14 nearly, and the secant 101 ; but the co-sine will be only 99. Then set the index on the angle 18 deg. and with the same radius of 100, the sine will be 31, the tangent $32\frac{1}{2}$, and the secant 105 ; but the co-sine will be only $94\frac{1}{4}$.—Now set the index on 45 deg. and to the same radius of 100, the sine and co-sine are each 70.7, the tangent and co-tangent are each 100, and the secant and co-secant are each 141.

191. In all circles, great or small, the sine of 90 deg. (called the sine total,) the tangent of 45 deg. the chord of 60 deg. and half the secant of 60 deg. are each equal to the radius which describes the circle.

192. The chord, sine, tangent, and secant of an arc in one circle is to the chord, sine, tangent, and secant of the same arc in another circle, as the radius of circle one is to the radius of the other.

193. In any plane triangle, the sides are proportional to the sines of their opposite angles.

194. In any right-angled triangle, the square of the longest side is equal to the sum of the squares of the other two sides.

195. In all plane triangles the sum of the three angles is equal to two right angles, or 180 deg.

196. The longest side of any triangle is opposite the greatest angle.

197. An angle in a semicircle is a right angle.

198. An angle in a segment less than a semicircle is greater than a right angle.

199. An angle in a segment greater than a semicircle is less than a right angle.

200. An angle at the circumference of a circle is half the angle at the centre, standing on the same arc; and it is measured by half the arc it subtends.

TRIGONOMETRY.

201. Trigonometry is the application of the properties of the circle to ascertain the unknown parts of triangles, some of the parts being given; and it is spheric, or plane.

202. In spheric trigonometry the sides of triangles are formed by the intersection of the arcs of three great circles.

203. In plane trigonometry the sides of triangles are formed by the meeting of three straight lines; and it is divided into right angled, and oblique angled.

204. In right-angled plane trigonometry, one of the angles is a right angle.

205. In oblique angled plane trigonometry all the angles are oblique.

206. Every plane triangle has six parts, viz. three sides and three angles; three of the parts must be given to find the other three; and one of the given parts must be a side.

207. If in a right-angled plane triangle one of the acute angles be given, all the angles are given—for the given acute angle being subtracted from 90 deg. the remainder will be the other acute angle; and the third angle is a right angle, or 90 deg.

208. Either side of a right-angled plane triangle may be made radius.

209. The longest side is called hypothenuse, the other sides are called legs, one base, the other perpendicular.

Application of the Diagram of Navigation to right-angled plane Trigonometry.

210. *Theorem.*—In any right-angled plane triangle, if the hypothenuse be made radius, one leg will be the sign of its opposite angle, and the other leg the co-sine of the same angle; but if either leg be made radius, the other leg will be the tangent of its opposite angle, and the hypothenuse will be the secant of the same angle.

211. Given a right-angled plane triangle, of which the hypothenuse is 217, the base 184, and the perpendicular 115—the angle opposite the leg 115 is 32 deg. and the angle opposite the leg 184 is 58 deg.—the angle opposite the hypothenuse being the right angle, or 90 deg. Let the hyp. 217 be made radius, and set the index on 32 deg. it is seen that the leg 115 is the sine of its opposite angle 32 deg. and that the leg 184 is the cosine of the same angle.

Then let the leg 184 be radius, and set the index on the angle 32 deg. it is seen that the other leg 115 is the tangent of its opposite angle 32 deg. and that the hyp. 217 is the secant of the same angle.

Now let the leg. 115 be made radius, and set the index on the angle 58 deg. it is seen that the other leg 184 is the tangent of its opposite angle 58 deg. and that the hyp. 217 is the secant of the same angle.

From the foregoing theorem we have the following cases in right-angled plane trigonometry.

CASE 1.

212. Given the hyp. 112 as radius, and a contiguous angle 30 deg. to find the legs. Set the index on the given angle 30 deg. and the hyp. 112 on the index will give the leg 56, being the sine of the angle 30 deg. and the leg 97 being the co-sine of the same angle.

CASE 2 & 3.

213. Given as radius one leg 78 and its opposite angle 26 deg. to find the other leg and the hypothenuse. The index set on the other acute angle 64 deg. the leg 78 as radius will give the other leg 160 as tangent—being opposite the angle 64 deg. and the hyp. 178 being the secant of the same angle.

CASE 4 & 5.

214. Given the hyp. 154 and one leg 118 as radius, to find the angles and the other leg. Set the index so that the hyp. 154 will meet the leg 118 as radius, it will be on the angle 40 deg. which is opposite the other leg 99 as tangent; then 40 deg. from 90 deg. gives the other acute angle 50 deg.

215. Given the legs 211 and 81, to find the angles

and the hypothenuse. Let the leg 211 be radius, and set the index to meet the other leg 81 as tangent—it will give the opposite angle 21 deg. and the hyp. 226 as secant of the same angle; and the other acute angle is 69 deg.

Problems which may be performed by the Diagram of Navigation.

216. Given the perimeter, or the sum of the four sides of a square, to find the diagonal.

Rule.—Set the index on 45 deg. the length of a side of the square taken on either side of the diagram, will give the diagonal on the index.

Example.—The perimeter of a square is 300—what is the diagonal?

Here $\frac{1}{4}$ of 300 is 75, the side of the square—then the index set on 45 deg. 75 from either side of the diagram, will give on the index the diagonal 106.

217. Given the sides of a rectangle to find the diagonal.

Rule.—Set the index to meet the two sides—and they will give on the index the diagonal.

Example.—The two sides of a rectangle are 158 and 84, what is the diagonal?

Here the index set to meet 158 with 84 will give on the index 179 the diagonal.

218. Given the hypothenuse and the sum of the legs, to find each leg and the angles.

Rule.—Set the index so that on it the hyp. shall meet two numbers equal to the given sum of the legs—those numbers will be the legs, and the index will be on one of the acute angles.

Example.—Of a right-angled plane triangle the hyp. is 50 and the sum of the legs is 70 ; required each leg and the angles.

Here the hyp. 50, on the index will meet 40 and 30 the legs, and the index will be on 37 deg. one of the acute angles—hence the other is 53 deg.

219. Given one of the legs, and the sum of the hyp. and the other leg, to find these sides separately and the angles.

Rule.—Set the index so that the sum of the hyp. and leg sought shall meet the given leg ; the two sides will be separately given, and the index will be on one of the acute angles.

Example.—Of a right-angled plane triangle one of the legs is 30, and the sum of the other leg and hyp. is 90 ; what are these sides separately, and the angles ?

Here the given leg 30 will give the hyp. 50, and the other leg 40 and the acute angles 37 deg. and 53 deg.

220. Given one of the legs and the difference between the hyp. and the other leg, to find these sides separately, and the angles.

Rule.—Set the index to meet the given leg, with the other leg equal to the hyp. less by the given difference; each side will be separately given and also the acute angles.

Example.—Of a right-angled plane triangle one leg is 30, and the difference between the hyp. and the other leg is 10; what are these sides separately, and the angles?

Here the leg 30, with the other leg equal to the hyp. less 10, will give that leg 40, and the hyp. 50, and the acute angles 37 and 53 deg.

221. Given the diagonal of a rectangle, and the sum of the sides, to find the sides.

Rule.—Set the index to meet the diagonal with two numbers equal to the given sum of the sides; these sides will be separately given.

Example.—The diagonal of a rectangle is 148, and the sum of the sides is 209; what is each side?

Here the diagonal 148 will meet 110 and 99, the sides required.

Theorems in right-angled plane trigonometry, from which are derived the solutions for calculating the problems in navigation, by logarithms.

222. *Theorem.*—In a right-angled plane triangle, if one side as radius be equal to the distance sailed, and a contiguous angle be equal to the course, the side adjacent to this angle will be equal to the difference of latitude, being the co-sine of said angle; and the side opposite will be equal to the departure, being the sine of the same angle. Hence these solutions in the first

case of plane sailing. As radius is to the distance, so is the co-sine of the course to the difference of latitude; and as radius is to the distance, so is the sine of the course to the departure.

Example.—Given the course 3 points, and distance 243 miles; what is the diff. lat. and departure?

As rad.	90°	10.00000
Is to dist.	243	2.38561
So is co-sine course 3 pts.		9.91985
To diff. lat.	202	2.30546

And as rad.	90°	10.00000
Is to dist.	243	2.38561
So is sine course	3 pts.	9.74474
To departure	135	2.13035

By inspection. In the table of triangles look under the course 3 pts. and opposite the distance 243 stands 202 the diff. lat. and 135 the departure.

223. *Theorem.*—In a right-angled plane triangle, if one side as radius be equal to the difference of latitude, and a contiguous angle be equal to the course; the side adjacent to this angle will be equal to the distance, being the secant of said angle; and the side opposite will be equal to the departure, being the tangent of said angle. Hence these solutions in the 2d case of plane sailing:

As rad. is to diff. lat. so is the secant of the course to the distance; and as rad. is to diff. lat. so is the tangent of course to the departure.

Example.—Given the diff. lat. 143 and the course $3\frac{1}{4}$ points; what is the distance and departure?

As rad.	45°	10.00000
Is to diff. lat.	143	2.15534
So is sec. course	$3\frac{1}{4}$ pts.	10.09517
To distance	178	2.25051

And as rad.	45°	10.00000
Is to diff. lat.	143	2.15534
So is tan. course	$3\frac{1}{4}$ pts.	9.87020
To the departure	106	2.02554

By inspection. Under the course $3\frac{1}{4}$ pts. and against the diff. lat. 143, stands the dist. 178 and the dep. 106.

224. *Theorem.*—In a right-angled plane triangle, if one side as radius be equal to the departure, and a contiguous angle be equal to the complement of the course, the side adjacent to this angle will be equal to the distance, being the secant of said angle, and the side opposite said angle will be equal to the difference of latitude, being the tangent of the said angle. Hence these solutions in the 3d case of plane sailing:

As rad. is to the dep. so is the co-secant of the course to the distance; and as rad. is to the dep. so is the co-tangent of the course to the diff. lat. The co-secant and the co-tangent of the course, are the same as the secant, and the tangent of the complement of the course.

Example.—Given the course $2\frac{1}{2}$ pts. and the dep. 116; what is the distance, and the diff. lat.?

As rad.	45°	10.00000
Is to dep.	116	2.06446
So is co-sec. of course	$2\frac{1}{2}$ pts.	10.32661
To distance	246	2.39107
And as rad.	45°	10.00000
Is to dep.	116	2.06446
So is co-tan. of course	$2\frac{1}{2}$ pts.	10.27204
To diff. lat.	217	2.33650

225. *Theorem.*—In a right-angled plane triangle if the hypothenuse as radius be equal to the distance, and one of the legs be equal to the difference of latitude, being the co-sine of the course, the angle formed by these sides will be equal to the course, and the side opposite this angle will be equal to the departure, being the sine of the course. Hence these solutions in the 4th case of plane sailing:

As the dist. is to rad. so is the diff. lat. to the co-sine of the course; and as rad. : dist. : : sine course : dep.

Example.—Given the distance 212 miles and diff. lat. 187; what is the course and departure?

As distance	212	2.32634
Is to rad.	90°	10.00000
So is diff. lat.	187	2.27184
To the co-sine of course	$2\frac{1}{2}$ pts.	9.94550
And as rad.	90°	10.00000
Is to dist.	212	2.32634
So is sine of course	$2\frac{1}{2}$ pts.	9.67339
To departure	100	1.99973

By inspection. In the table the distance 212 with the diff. lat. 187, will give the course $2\frac{1}{2}$ pts. and the dep. 100 miles.

226. *Theorem.*—In a right-angled plane triangle if the hypothenuse as radius be equal to the distance and one of the legs be equal to the departure,—the angle formed by these sides will be equal to the complement of the course, and the side opposite this angle will be equal to the difference of latitude. Hence these solutions in the 5th case of plane sailing :

As dist. is to rad. so is dep. to sine of the course ; and as rad. is to distance, so is the co-sine of course to diff. lat.

Example.—Given the distance 104 miles and dep. 64 ; what is the course and diff. lat. ?

As distance	104	2.01703
Is to rad.	90°	10.00000
So is departure	64	1.80618
To the sine of course or $3\frac{1}{4}$ points nearly.	38°	9.78915
And as rad.	90°	10.00000
Is to the distance	104	2.01703
So is the co-sine course or $3\frac{1}{4}$ points nearly	38°	9.89653
To diff. lat. 82		1.91356

By inspection. In the table, the distance 104 with the departure 64, give the course 38 deg. and diff. lat. 82 miles.

227. *Theorem.*—In a right-angled plane triangle if one of the sides as radius be equal to the difference of latitude, and another side as tangent be equal to the departure,—the angle opposite the side as tangent will be equal to the course, and the side adjacent to this

angle will be equal to the distance, being the secant of said angle. Hence these solutions in the 6th case of plane sailing :

As diff. lat. is to rad. so is dep. to tangent of the the course. And as rad. is to diff. lat. so is secant of the course to the distance.

Example.—Given the diff. lat. 89 and dep. 36; what is the course and distance?

As diff. lat.	89	1.94939
Is to rad.	45°	10.00000
So is dep.	36	1.55630
To tangent of course or 2 pts. nearly.	22°	9.60691
And as rad.	45°	10.00000
Is to diff. lat.	89	1.94939
So is secant of course	22°	10.03283
To the distance	96	1.98222

By inspection. In the table the diff. lat. 89 with the dep. 36 give the course 22 deg. and distance 96 miles.

228. *Theorem.*—In a right-angled plane triangle if the hypothenuse as radius be equal to the difference of longitude, and a contiguous angle be equal to the parallel of latitude sailed in—the side adjacent this angle will be equal to the meridian distance, being the cosine of said angle.—Hence this solution in the 1st case of parallel sailing :

As rad. is to the diff. lon. so is the co-sine of the lat. to the meridian distance.

Example.—Given the diff. lon. 140 miles and the lat. 45 deg. what is the meridian distance?

As rad.	90°	10.00000
Is to the diff. lon.	140	2.14613
So is the co-sine of lat.	45°	9.84949
To the meridian dist.	99	1.99562

By inspection. In the table under 45 deg. the diff. lon. 140, taken in the dist. column, gives in the lat. column the meridian distance 99 miles.

229. *Theorem.*—In a right-angled plane triangle if one side as radius be equal to the meridian distance, and a contiguous angle be equal to the parallel of latitude—the side adjacent to this angle will be equal to the diff. lon. being the secant of said angle. Hence this solution in the 2d case of parallel sailing:

As rad. is to the mer. distance so is the secant of the latitude to the diff. of longitude.

Example.—Given the meridian distance 110 miles and the lat. 36 deg.; what is the diff. longitude?

As rad.	45°	10.00000
Is to the mer. dist.	110	2.04139
So is the secant of lat.	36°	10.09204
To the diff. of long.	136	2.13343

By inspection. In the table under lat. 36 deg. the mer. distance 110 taken in the lat. column gives the diff. lon. in the dist. column 136.

230. *Theorem.*—In a right-angled plane triangle if the hypothenuse as radius be equal to the diff. of longitude, and the side adjacent be equal to the meridian distance—the angle formed by these two sides will be equal to the latitude, being its co-sine. Hence this solution in the 3d case of parallel sailing:

As diff. lon. is to rad. so is meridian distance to the co-sine of the latitude.

Example.—Given the diff. lon. 120 miles, and the meridian distance 105; what is the latitude?

As diff. lon.	120	2.07918
Is to rad.	90°	10.00000
So is meridian dist.	105	2.02119
To co-sine of lat.	28°57′	9.94201

By inspection. In the table the diff. lon. 120 taken in the dist. column, with the meridian distance 105 taken in lat. column, give the lat. 29 degrees.

231. *Theorem.*—In a right-angled plane triangle, if the hypothenuse as radius be equal to the difference of longitude, and a contiguous angle be equal to the middle latitude; the side adjacent to this angle will be equal to the departure (or meridian distance,) being the co-sine of said angle. Hence this solution in the 1st case of middle lat. sailing—to find the departure.

As rad. is to diff. lon. so is co-sine of middle lat. to the departure.

Example.—Given the diff. lon. 115 miles, and the mid. lat. 43 deg.; what is the departure?

As rad.	90°	10.00000
Is to diff. lon.	115	2.06070
So is co-sine mid. lat.	43°	9.86413
To the departure	84	1.92483

By inspection. In the table under 43 deg. the mid. lat. the diff. lon. 115 taken in the dist. column will give the dep. 84 in the column marked lat.—If the *complement* of the mid. lat. be used, then the departure is found in the column marked dep.

232. *Theorem.*—In a right-angled plane triangle if one side as radius be equal to the departure, and a contiguous angle be equal to the middle latitude, the side adjacent to this angle will be equal to the difference of longitude, being the secant of the said angle. Hence this solution in the 2d case of middle latitude sailing, to find the difference of longitude:

As rad. is to the dep. so is the secant of the mid. lat. to the diff. lon.

Example.—Given the dep. 104 miles and the mid. lat. 43 deg. 30 min.; what is the diff. longitude?

As rad.	45°	10.00000
Is to dep.	104	2.01703
So is secant mid. lat.	43°30′	10.13944
To diff. long.	143½	2.15647

By inspection. The tables are calculated to whole degrees only—therefore under lat. 43 deg. the dep. 104, taken in the lat. column, gives, in the dist. column, the diff. long. 142; and under lat 44 deg. it gives 145: the mean of these sums is 143½ the true diff. lon.

233. Theorem. In a right-angled plane triangle if one side as radius be equal to the meridional difference of latitude, and another side as tangent be equal to the difference of longitude, the angle opposite the side as tangent will be equal to the course. Hence this solution in the first case of Mercator's sailing, to find the course:

As mer. diff. lat. is to rad. so is diff. lon. to tangent of the course.

Example.—Given the mer. diff. lat. 202 and the diff. lon. 125; what is the course?

As mer. diff. lat.	202	2.30535
Is to rad.	45°	10.00000
So is diff. lon.	125	2.09691
To tan. of course	31° 45′	9.79156

By inspection. In the table look for mer. diff. lat. 202 in the lat. column to agree with diff. lon. 125 in the dep. column the course will be given at top or bottom.—The nearest numbers in the table in this case are 201.8 and 126.1 which give the course 32 deg.

234. *Theorem.*—In a right-angled plane triangle if one side as radius be equal to the meridional difference of lat. and a contiguous angle be equal to the course, the side opposite this angle will be equal to the difference of longitude, being the tangent of said angle. Hence this solution in the 2d case of Mercator's sailing, to find the difference of longitude :

As rad. is to mer. diff. lat. so is the tangent of course to the diff. lon.

Example.—Given the mer. diff. lat. 172 and course 3 pts.; what is the diff. longitude ?

As rad.	45°	10.00000
Is to mer. diff. lat.	172	2.23553
So is tan. course	3 pts.	9.82489
To the diff. long.	115	2.06042

By inspection. In the table under the course 3 pts. the mer. diff. lat. 172 taken in the lat. column gives, in the dep. column 115 for diff. long.

235. As the proper difference of latitude is to the departure, so is the meridional difference of latitude to the difference of longitude.

Example.—Given the proper diff. lat. 160, the dep. 125 and the mer. diff. lat. 320; what is the diff. longitude ?

As 160 is to 125 so is 320 to 250. Or,

As prop. diff. lat.	160	2.20412
Is to departure	125	2.09691
So is mer. diff. lat.	320	2.50515
		4.60206
To diff. lon.	250	2.39794

236. *Theorem.*—In a right-angled plane triangle, if one side as radius be equal to the difference of longitude, and a contiguous angle be equal to the complement of the course; the side opposite this angle will be equal to the meridional difference of latitude, being the tangent of said angle. Hence this solution:

Rad. : diff. lon. :: co-tang. course : mer. diff. lat.

Example.—Given the course 38° and diff. lon. 125 miles; what is the mer. diff. lat.?

Rad.	90°	10.00000
Is to diff. lon.	125	2.09691
So is co-tang. course	38°	10.10719
To mer. diff. lat.	160	2.20410

By inspection. Under the course 38° and opposite the diff. lon. 125 taken in the dep. col. stands in the lat. col. 160 the mer. diff. lat.

237. A table of the rules collected, by which the problems in plane, parallel, middle latitude, and Mercator's sailing may be calculated by logarithms.

In Plane Sailing.

GIVEN.	REQUIRED.	SOLUTIONS.
Course and Distance.	Diff lat. and Departure.	Rad. : dist. :: co-sine course : diff. lat. Rad. : dist. :: sine course : dep
Course and Diff. lat.	Distance and Departure.	Rad. : diff. lat. :: sec. course : dist. Rad. : diff. lat. :: tang. course : dep.
Course and Departure.	Distance and Diff. lat.	Rad. : dep. :: co-sec. course : dist. Rad. : dep. :: co-tang. course : diff. lat.
Distance and Diff. lat.	Course and Departure.	Dist. : rad. :: diff. lat. : co-sine course. Rad. : dist. :: sine course : dep.
Distance and Departure.	Course and Diff. lat.	Dist. : rad. :: dep. : sine course. Rad. : dist. :: co-sine course : diff. lat.
Diff. lat. and Departure.	Course and Distance.	Diff. lat. : rad. :: dep. : tang. course. Rad. : diff. lat. :: sec. course : dist.

In Parallel Sailing.

Latitude and Diff. lon.	Meridian distance.	Rad. : diff. lon. :: co-sine lat. : mer.dist.
Latitude and Mer. dist.	Diff. lon.	Rad. : mer. dist. :: sec. lat. : diff. lon.
Diff. lon. and Mer. dist.	Parallel of latitude.	Diff. lon. : rad. :: mer. dist. : co-sine lat.

In Middle Latitude Sailing.

Diff. lon. and Mid. lat.	Departure.	Rad. : diff. lon. :: co-sine mid. lat. : dep.
Departure & Mid. lat.	Diff. lon.	Rad. : dep. :: sec. mid. lat. : diff. lon.
Diff. lat. Diff. lon. and Mid. lat.	Course.	Diff.lat. : diff.lon. :: co-sine mid.lat. :tang. course, or diff. lat. : rad. :: dep. : tang. course.

In Mercator's Sailing.

Mer. diff lat. and diff. lon.	Course.	Mer. diff. lat. : rad. :: diff. long. : tang. course.
Course and Mer. diff lat.	Diff. lon.	Rad. : mer. diff. lat. : tang. course : diff. lon.
Prop. diff. lat. Dep. and Mer. diff. lat.	Diff. lon.	Prop. diff. lat. : dep. :: mer. diff. lat. : diff. lon.
Course and Diff. lon.	Mer. diff. lat.	Rad. : diff. lon. :: co-tang. course : mer. diff. lat.

238. In the preceding table are to be found solutions for all the usual cases throughout the different methods of sailing. There are other proportions which give the same results. As far as practicable I have selected those in which radius is the first term of the proportion, which renders the operations by logarithms more easy for the learner.

The solutions in middle latitude and in Mercator's sailing, which are common with plane sailing, are not repeated. It is presumed the omission will not be considered a deficiency, as the table contains every requisite analogy; and the learner will readily discover that the methods of middle latitude and Mercator's, vary from plane sailing only in relation to the longitude.

239. Estimating the difference of longitude by the middle latitude is the usual practice in ordinary voyages. It is convenient, and sufficiently accurate for short runs, or a day's work in common latitudes; but in high latitudes, when the distance embraces distant parallels, it is erroneous.

240. Mercator's sailing is perfectly accurate, and the problems in it are easily performed by the diagram of navigation, the meridional parts being readily obtained by the arc of expanded degrees of latitude, or taken from a table when great exactness is required.

*General principles of the Tides.**

241. "A tide is that motion of the water in the seas and rivers, by which they are found to rise and fall in a regular succession; and this flowing and ebbing is caused by the attraction of the sun and moon."

242. "The parts of the earth directly under the moon, or where the moon is in the zenith; and those places which are directly opposite to the former, or under the nadir, will have high water at the same time."

243. "Those parts of the earth where the moon appears in the horizon, or 90° distant from the zenith and nadir will have ebb or low water."

* Extracted from Keith's Treatise on the Globes.

244. "The time of high water is not precisely at the time of the moon's coming to the meridian, but about an hour after."

245. "The tides are greater than ordinary twice every month, viz: at the times of new and full moon, and these are called *spring tides.*"

246. "The tides are less than ordinary twice every month; that is about the time of the first and last quarters of the moon, and these are called *neap tides.*"

247. "The spring tides do not happen exactly on the day of the change or full moon, nor the neap tides exactly on the days of the quarters, but a day or two afterwards."

248. "When the moon is nearest to the earth, or in perigee, the tides increase more than in similar circumstances at other times."

249. "The spring tides are greater a short time before the vernal equinox, and after the autumnal equinox, viz: about the latter end of March and September, than at any other time of the year."

250. "Lakes are not subject to tides; and small inland seas, such as the Mediterranean and Baltic, are little subject to tides. In very high latitudes, north or south, the tides are also inconsiderable."

251. "The time of the tides happening in particular places, and likewise their height may be very different according to the situation of these places; because the motion of the tides is propagated swifter in the open sea, and slower through narrow channels or shallow places; and being retarded by such impediments, the tides cannot rise so high."

252. "By the general theory the motion of the tides ought always to follow the moon, and flow from east to west; but to allow the tides their full motion the ocean in which they are produced ought to extend from east to west at least 90° or 6255 English miles; because that is the distance between the places where the water is the most raised and depressed by the moon."

253. "Hence it appears that it is only in the great oceans that the tide can flow regularly from east to west; and hence we also see why the tides in the torrid zone, between Africa and America, though nearly under the moon, do not rise so high as in the temperate zones northward and southward, where the ocean is considerably wider."

254. "The tides in the Atlantic, in the torrid zone, flow from east to west till they are stopped by the Continent of America; there the trade winds likewise continue to blow in that direction."

255 "When the action of the moon upon the waters has in some degree ceased, the force of the trade winds in a great measure, prevents their return toward the African shores. The water thus accumulated in the Gulf of Mexico returns to the Atlantic between the Island of Cuba, the Bahama Islands and East Florida, and forms that remarkable strong current called the Gulf of Florida," or the *Gulf Stream*.

256. *A table of the velocity of the Wind.*

Miles per hour	Common appellations.
1	Hardly perceptible.
2 to 3 .	Just perceptible.
4 to 5 .	Gentle pleasant wind.
10 to 15 .	Pleasant brisk gale.
20 to 25 .	Very brisk.
30 to 35 .	High winds.
40 to 45 .	Very high.
50	A storm, or tempest.
60	A great storm.
80	A hurricane.
100	A hurricane that tears up trees, buildings, &c.